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A parametric analysis of reduced order models of viscous flows in turbomachinery ☆

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Abstract

The unsteady flow created by a cascade of oscillating airfoils is investigated. A frequency-domain model is constructed and the proper orthogonal decomposition technique is applied to construct several reduced order models of subsonic and transonic flows. A cascade of NACA-5506 airfoils is numerically investigated to show that reduced order models with only 35 degrees of freedom accurately predict the unsteady response of the full model with approximately 6000 degrees of freedom in the subsonic regime, for a broad range of Mach numbers. In the transonic regime, 55 degrees of freedom were shown to be required to accurately predict the response of the full model. The increased number of degrees of freedom is shown to be due to the presence of the shock and not the increase in the Mach number per se.

1. Introduction

The dynamics of unsteady flows and fluid–structure interaction phenomena may be well predicted by use of computer fluid dynamics (CFD) codes using a large number of degrees of freedom, up to millions. Due to recent advances in computer and software engineering, the computation time required to perform such accurate calculations has been reduced, but remains prohibitive when numerous analyses, such as parametric analyses, are required. In addition, the large CFD calculations are practically impossible to use directly in control applications because most control strategies are based on a low number of state variables, ranging typically up to 100. One technique designed to overcome the limitations generated by the large number of degrees of freedom in CFD calculations is the use of reduced order models. Generally, a reduced order model (ROM) is a simplified model which has a dramatically lower number of degrees of freedom than the original model and it is capable to predict accurately the dynamics of the original model.

Although developing reduced order models has regained momentum only recently, there are several classical techniques which fall under this category of modelling. For example, these techniques have been used to model fluid–structural systems encountered in aeroelasticity in the early attempts of model reduction. In that field, studies such as the actuator disk theory (Greitzer, 1976), and other phenomenological studies (Moore and Greitzer, 1986) are based mostly on physical insights. These insights are useful, but they are limited to a small range of parameter variations in the modelled system, such as forcing frequency or static and dynamic loads, among others. More recent analyses (Peterson and Crawley, 1988; Ueda and Dowell, 1984) use *data-derived* reduced order models, such as Padé approximants (Feldmann and Freund, 1995; Karpel, 1999) or describing functions by curve fitting unsteady aerodynamical transfer functions. Furthermore, eigenmode summation techniques in either time or frequency domains (Dowell, 1995; Hall, 1994), and proper orthogonal decomposition, also known as the Karhunen–Loève method (Epureanu et al., 1999, 2000, 2001b; Kim, 1998; Romanowski, 1996) have been successfully used as well. Most of these techniques are linear, while a

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few focus on nonlinear systems (Kappagantu and Feeny, 1999; Silva, 1999; Silva and Bennett, 1993, 1995). Reduced order models have been constructed been used primarily in the time domain (Pettit and Beran, 2002; Willcox et al., 2001, 2002) and applied to a variety of aeroelastic problems including control (Lucia et al., 2002; Willcox et al., 1999) and design (LeGresley and Alonso, 2000). Recently, Dowell and Hall (2001) have prepared a review of eigenmode and proper orthogonal decomposition based ROMs for linear and nonlinear aerodynamics that summarizes and discusses much of the relevant literature and recent research advances.

Although most recent research on reduced order modelling has focused on time linearized systems, these linearized techniques have been used by Noor and Stone to model both linear and *nonlinear* phenomena (Noor 1994; Stone and Cutler, 1996). The linearized reduced order modelling technique has been applied to a variety of problems, such as Burger's model of turbulence (Canuto et al., 1988; Sirovich, 1987a, b), Euler equations, Navier–Stokes equations (Deane et al., 1988), Raleigh–Bénard convection (Holmes et al., 1996), and boundary layer models (Liu et al., 1994). As distinct from *linearized* analyses, *fully nonlinear* normal modes and reduced order models have been investigated also for low-dimensional systems (Shaw and Pierre, 1994). However, the nonlinear normal modes were shown to be very complex for high-dimensional systems. Also, internally balanced reduced order models (Laub et al., 1987; Moore, 1981; Willcox and Peraire, 2002) have been used in control applications (Baker et al., 1996). Holmes et al. (1996) and Sirovich (1987a, b) used proper orthogonal decomposition (POD) in the context of turbulent flows as a technique which allows for the identification of naturally forming coherent structures from numerical simulations or experiments. These coherent structures contain most of the energy and are usually the most important components of the dynamics (Feeny, 2002; Kappagantu and Feeny, 2000a, b).

Proper orthogonal decomposition (POD) is a technique used for model reduction because it allows one to obtain good approximations of the spatial modes of vibration of a system using the response of the system to various excitations. The POD technique has been first used by experimentalists to analyze test data and recently has been applied to a wider variety of problems, such as wind load calculations (Bienkiewicz, 1996; Uematsu et al., 1997) and coherent structure identification (Holmes et al., 1996; Sirovich, 1987a). The crucial assumption made in the POD technique is that the dynamics of large systems is in fact low dimensional (Georgiou and Schwartz, 1996), i.e., the inertial manifold of the dynamics is low dimensional. For a large category of problems, this assumption holds because in many cases most of the energy of the system being analyzed is contained in the dynamics of a few modes. The trade-off between accuracy and complexity of ROMs may be estimated (Slater et al., 2002), and is determined by the dimension of the inertial manifold for each particular application.

Reduced order modelling has been used in the time and frequency domains for both unsteady analyses and ROM construction (Epureanu et al., 1999, 2000, 2001b). Inviscid flows have been investigated using the full potential equation and POD techniques in the frequency domain (Epureanu et al., 2001a). These studies have shown that *inviscid* unsteady flows passing through a two-dimensional turbomachinery cascade may be modelled accurately using ROMs with approximately 25 degrees of freedom. Also, the required number of modes in a ROM has been shown to be only weakly dependent on the upstream far-field Mach number, while the critical factor is the accuracy of the POD modes used for model reduction. The value of the Mach number for *inviscid* flows was shown to have an important influence on the accuracy of the POD modes and to influence indirectly the accuracy of the ROMs (Epureanu et al., 1999, 2000, 2001a, b).

In this paper, we apply the POD technique in the frequency domain and construct ROMs of unsteady *viscous* flows in a turbomachinery cascade. The problem of a forced excited flow is investigated. A coupled viscous-inviscid nonlinear model of subsonic and transonic flows in a cascade of airfoils is used to compute the steady flow. Then, a small amplitude motion of the airfoils about their steady flow configuration is considered. The unsteady flow is linearized about the nonlinear steady response based on the observation that in many cases the unsteadiness in the flow has a substantially smaller magnitude than the steady component. A generic compressor cascade geometry is considered. The airfoils in the cascade are NACA-5506 and have a chord c. The gap between the airfoils is denoted by G. A solidity G/c of value 1 is considered. The stagger angle γ is 45°. The inflow angle Θ is 55°, and the Reynolds number based on chord and upstream velocity Re is 5×10^5. This configuration is numerically investigated for several cases, i.e., subsonic cases where the upstream far-field Mach number is 0.5, 0.6 and 0.7, and transonic cases where the upstream far-field Mach number is 0.8 and 0.9.

The dependency of the required number of aerodynamic modes in ROMs of *viscous* flows on one of the most significant parameters of the system, the far-field upstream Mach number, is investigated. For *inviscid* flows, the dimension of the ROMs has been shown to be strongly dependent on the accuracy of the POD modes used for model reduction and weakly dependent on the Mach number per se (Epureanu et al., 2001a). Distinct from *inviscid* flows, for *viscous* flows it has been observed that transonic ROMs require a larger number of modes than the subsonic ROMs for a similar geometry, range of reduced frequencies and interblade phase angles (Epureanu et al., 1999, 2000, 2001b). The increased number of modes may be due to viscous effects at increased Mach number per se, or the viscous effects in the vicinity of the strong spatial gradients in the region of shocks (for transonic flows). These two possible causes are investigated.

The ROM obtained using snapshots computed at an upstream far-field Mach number M of 0.5 are also used to construct ROMs of flows with a Mach number of 0.45. In such case, the accuracy of the ROMs is found to be dependent on the difference in the Mach number values for the flow and the snapshots used in the POD technique. Trends in the dynamics of the system are shown to be inadequately predicted even for small changes in the steady Mach number, e.g., cut-on or cut-off frequencies, magnitudes of the coefficient of lift.

2. Flow modelling and model reduction

Typical flows of interest in turbomachinery are characterized by a large Reynolds number of order of a million, and the effects of the flow viscosity is concentrated in a thin region around the solid boundaries and the wake, also known as a boundary or inner layer. Thus, the flow is decomposed into an inviscid outer flow and a viscous inner flow. In the following, we briefly outline the main characteristics of the model used. This model has been thoroughly validated and discussed for both subsonic and transonic cases and the details may be found in Epureanu et al. (2000, 2001b). The present paper is focused on the influence of the Mach number on the dimension of the reduced order models constructed using proper orthogonal decomposition.

The upstream far-field Mach number affects the viscous as well as the inviscid flow. The inviscid flow is modelled by the full potential equation, which may be expressed as

$$\nabla^2 \phi = \frac{1}{c^2}\left[\frac{\partial^2 \phi}{\partial t^2} + 2\nabla\phi \cdot \nabla\frac{\partial \phi}{\partial t} + \frac{1}{2}\nabla\phi \cdot \nabla(\nabla\phi)^2\right], \tag{1}$$

where c is the local speed of sound given by

$$c^2 = c_0^2 - (\gamma - 1)\left[\frac{\partial \phi}{\partial t} + \frac{1}{2}(\nabla\phi)^2\right], \tag{2}$$

with c_0 denoting the stagnation speed of sound and γ the ratio of specific heats. The full potential is written in a conservative form as

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho\nabla\phi) = 0, \tag{3}$$

where ρ is the flow density, which is expressed in terms of the potential as

$$\rho = \rho_T\left\{1 - \frac{(\gamma - 1)}{c_0^2}\left[\frac{\partial \phi}{\partial t} + \frac{1}{2}(\nabla\phi)^2\right]\right\}^{1/(\gamma-1)}. \tag{4}$$

The potential is discretized using a Galerkin weighted-residual finite element method. An integral formulation is used in the form

$$\int\int_D \left[\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho\nabla\phi)\right] N_i \,\mathrm{d}x\,\mathrm{d}y + \oint \left(Q - \rho\frac{\partial \phi}{\partial \mathbf{n}}\right) N_i \,\mathrm{d}\xi = 0, \tag{5}$$

where Q is the prescribed mass flux on the boundary, ξ is the distance along the boundary, N_i are shape functions, and $\mathbf{n}$ is the local normal direction at the boundary of the domain over which the full potential equation is solved.

The Galerkin formulation is used to solve both the steady and the unsteady problems. The small disturbance assumption is used based on the observation that the unsteadiness in the flow is much smaller than the steady background flow. Thus, a dynamically linearized unsteady problem is solved. The potential is decomposed into a steady value Φ and an unsteady small disturbance potential φ periodically varying in time, such that

$$\phi(x, y, z, t) = \Phi(x, y, z) + \Re[\varphi(x, y, z)\mathrm{e}^{\mathrm{j}\omega t}], \tag{6}$$

where $\varphi \ll \Phi$, j is the imaginary unit $\sqrt{-1}$, and $\Re$ denotes the real part.

Fig. 1 shows the domain where the inviscid outer flow is solved and the regions where different boundary conditions apply. On the airfoil boundary, Q is a flux which arises from the motion of the airfoil and the thickening of the viscous boundary layer (Epureanu et al., 2001b; Hall, 1994). The periodicity on the upstream region reads $\phi_{\mathrm{up}} = \phi_{\mathrm{down}}\mathrm{e}^{\mathrm{j}\sigma}$, where σ is the interblade phase angle. The wake boundary condition states that the jump in pressure across the wake is zero. Because the computation domain is not perfectly aligned with the wake, an additional injection flux is applied on the wake boundaries (Epureanu et al., 2000). For the steady problem, the upstream boundary condition specifies the

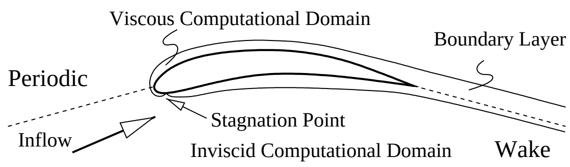

Fig. 1. Solution domain and boundary conditions.

value of the potential, while the downstream boundary condition specifies the downstream flux. For the unsteady problem, the boundary conditions are exact nonreflecting (Hall et al., 1993) for the linearized unsteady problem.

The boundary layer equations may be obtained performing a scale analysis under the assumption of a very large Reynolds number. In such an analysis, the diffusion process parallel to a body surface and wake may be neglected and the momentum equation normal to the surface may be replaced by the condition of zero normal pressure gradient throughout the boundary layer. In this analysis the local airfoil and wake curvature effects are neglected along with the local deformation of the airfoil profile.

The two unsteady compressible equations which describe the mass and momentum equations for the flow in the thin boundary layer shown in Fig. 1 are formally integrated to obtain the von Kármán energy and momentum integral equations. Also, laminar-turbulent transition is of considerable practical interest because it strongly influences where separation occurs. The e^n method to determine the transition is used. This method correlates the position of the transition to the position where the overall maximum amplification of Tollmien–Schlichting disturbances is e^n. We used the approximate spatial amplification curve derived by Drela (1986) based on the Orr–Somerfeld equation applied to a Falkner–Skan profile family (Epureanu et al., 2001b).

The solution domain used to compute the viscous flow is shown in Fig. 1, where the thickness of the domain is considered small in comparison to the chord. The steady integral boundary layer equations are parabolic in space so that boundary conditions have to be applied only at the stagnation point. Close to the stagnation point, the flow is similar to a flow over a wall. There is an analytical similarity solution for this flow that relates the displacement thickness to the inviscid tangential velocity. This similarity solution is used as boundary condition at the stagnation point.

The "snapshot" proper orthogonal decomposition method is used for model reduction. In this approach, the response of the linearized system with L degrees of freedom is obtained and stored in a solution vector $\mathbf{\Phi}_i$, for a set of N excitation frequencies ω_i. Each solution vector $\mathbf{\Phi}_i$ has L complex entries and contains both the phase and the magnitude of the response. A correlation matrix is formed and the modes containing the largest components of the energy of the dynamics are retained, following the proper orthogonal decomposition methodology described in detail in Epureanu et al. (2000, 2001b).

3. Numerical results

The models constructed are used to predict the flow state, while considering the motion of the airfoils (their reduced frequency and interblade phase angle) as inputs. The analysis is focused on determining the dimension of the flow dynamics, and the influence that the Mach number has on that dimension. Surely, the model of the flow may be considered to have as inputs the Mach number and the geometrical characteristics of the cascade as well. However, such an approach would not elucidate the fundamental physical question posed herein, i.e., what is the influence of the Mach number on the number of dominant modes in the dynamics. The proposed approach is to construct reduced order models for flows at certain Mach numbers and investigate their accuracy as applied to flows at other Mach numbers, thus estimating the strength of the dependence of the reduced order models on the Mach number.

A generic compressor cascade geometry composed of NACA-5506 airfoils of chord c is considered. The solidity G/c of the cascade has value 1, with G being the gap between the airfoils. The stagger angle γ is 45°. The inflow angle Θ is 55°, and the Reynolds number based on chord and upstream velocity Re is 5×10^5. This configuration is numerically investigated for several cases where the upstream far-field Mach number is 0.5, 0.6, 0.7, 0.8 and 0.9. The airfoils are assumed to vibrate with small amplitudes at reduced frequency $k = \omega c / v_\infty$ in a pitch motion about the mid-chord point. The interblade phase angle of the vibration of the airfoils is denoted by σ.

The proper orthogonal method is applied to a set of 700 snapshots obtained at 35×20 distinct values of the interblade phase angle σ (varying from $-180°$ to $180°$) and reduced frequency k (varying from 0 to 2). Six distinct Mach numbers are investigated, i.e., 0.45, 0.5, 0.6, 0.7, 0.8, and 0.9. The numerical results presented show the coefficient of lift acting on the vibrating airfoils. The interblade phase angle is maintained constant at a value of 90° for the computations where the reduced frequency k is varied. Similarly, the reduced frequency k is maintained constant at a value of 1 for the computations where the interblade phase angle σ is varied.

Fig. 2 shows the imaginary (in phase) and real (out of phase) parts of the coefficient of lift as a function of the reduced frequency k. The upstream far-field Mach number is 0.5. An accurate reduced order model may be constructed using only 35 modes. This model is accurate for various reduced frequencies, as well as various interblade phase angles, as shown in Fig. 3.

A similar number of degrees of freedom are required for a larger upstream far-field Mach number also. Fig. 4 shows the real and imaginary parts of the coefficient of lift for an upstream far-field Mach number of 0.6. An accuracy similar to the model obtained at Mach 0.5 is observed. This accuracy is maintained for various reduced frequencies, as well as various interblade phase angles, as shown in Fig. 5.

Further increasing the upstream far-field Mach number to 0.7 and 0.8 does not require an increased number of degrees of freedom for obtaining a similar accuracy. Fig. 6 shows the unsteady coefficient of lift at an upstream far-field Mach number of 0.7 and indicates that a 35 mode model is accurate for various reduced frequencies. The same 35 mode model is used successfully at various interblade phase angles as well (Fig. 7).

The steady flow field has a small transonic region for an upstream far-field Mach number of 0.8. The maximum Mach number in the field is 1.12 indicating the presence of a mild shock. Nevertheless, the number of degrees of freedom required to construct an accurate reduced order model is approximately the same as is for the subsonic flows discussed above. Fig. 8 shows that a 35 mode model is accurate for various reduced frequencies. Similarly, Fig. 9 shows that the accuracy of the model is maintained for various interblade phase angles σ as well.

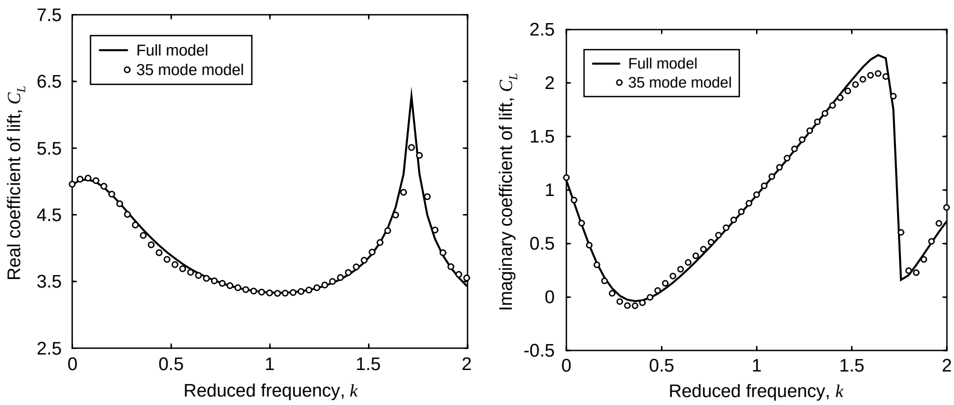

Fig. 2. Unsteady C_L obtained using the full model and a ROM at a Mach number of 0.5 and an interblade phase angle σ of 90°.

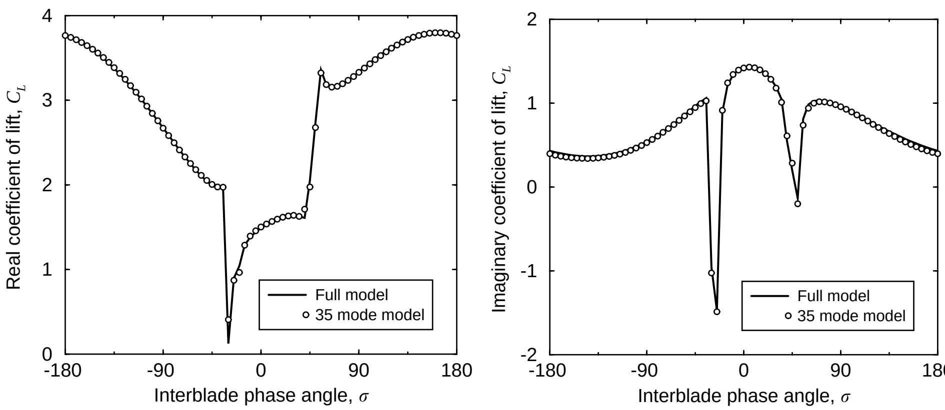

Fig. 3. Unsteady C_L obtained using the full model and a ROM at a Mach number of 0.5 and a reduced frequency k of 1.

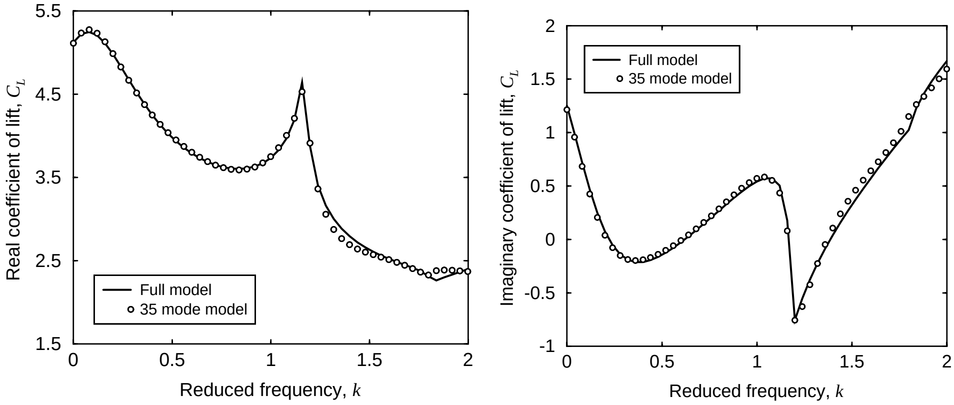

Fig. 4. Unsteady C_L obtained using the full model and a ROM at a Mach number of 0.6 and an interblade phase angle σ of 90°.

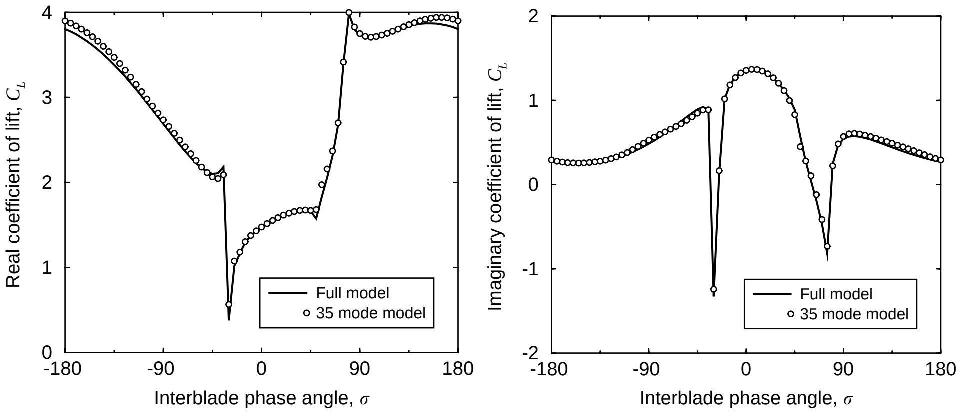

Fig. 5. Unsteady C_L obtained using the full model and a ROM at a Mach number of 0.6 and a reduced frequency k of 1.

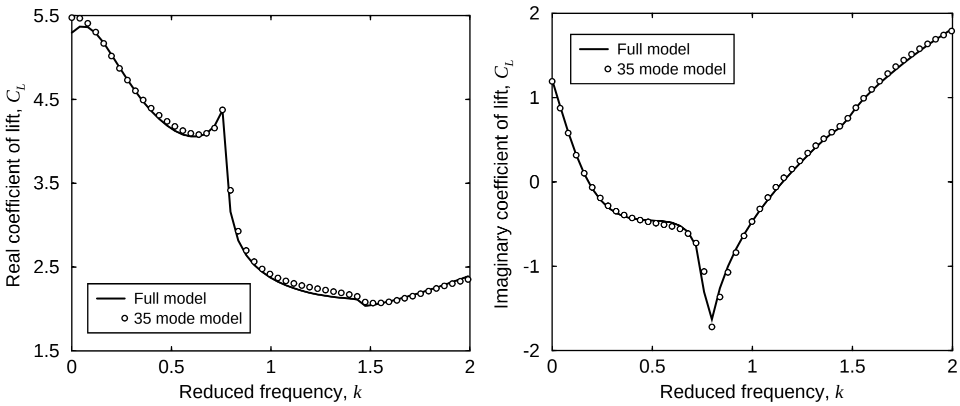

Fig. 6. Unsteady C_L obtained using the full model and a ROM at a Mach number of 0.7 and an interblade phase angle σ of 90°.

Nevertheless, the number of degrees of freedom required for similar accuracy at an upstream far-field Mach number of 0.9 is higher. Fig. 11 shows that a 35 mode model may exhibit different characteristics than the full model for certain reduced frequencies. A similar behavior is observed for certain interblade phase angles, as shown in Fig. 10.

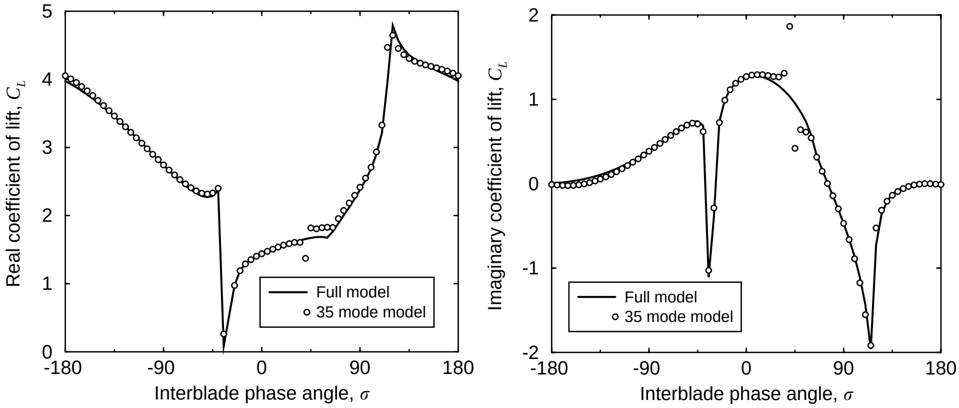

Fig. 7. Unsteady C_L obtained using the full model and a ROM at a Mach number of 0.7 and a reduced frequency k of 1.

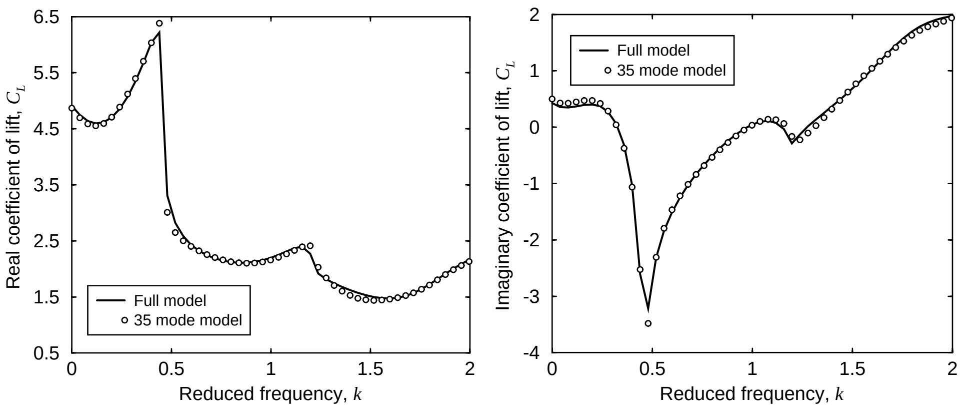

Fig. 8. Unsteady C_L obtained using the full model and a ROM at a Mach number of 0.8 and an interblade phase angle σ of 90°.

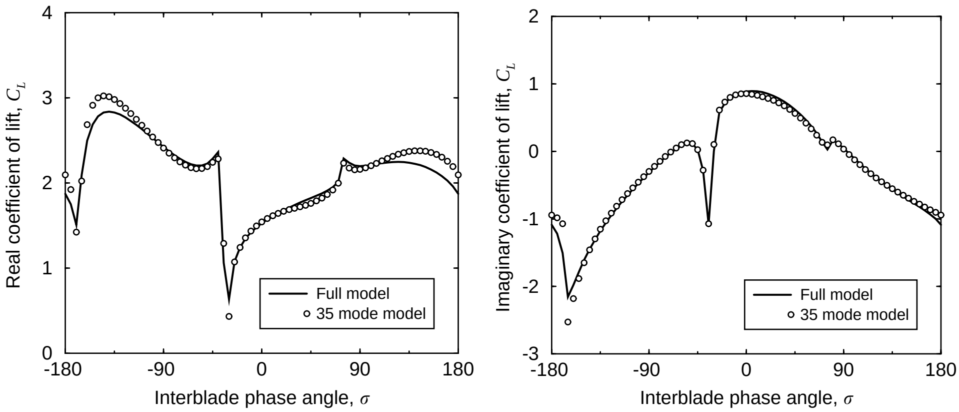

Fig. 9. Unsteady C_L obtained using the full model and a ROM at a Mach number of 0.8 and a reduced frequency k of 1.

To obtain similar accuracy, the number of degrees of freedom of the reduced order model of transonic flow is increased to 55. Fig. 11 shows the real and imaginary coefficient of lift obtained using a 55 mode model. Similar accuracy is observed for various interblade phase angles, as shown in Fig. 10.

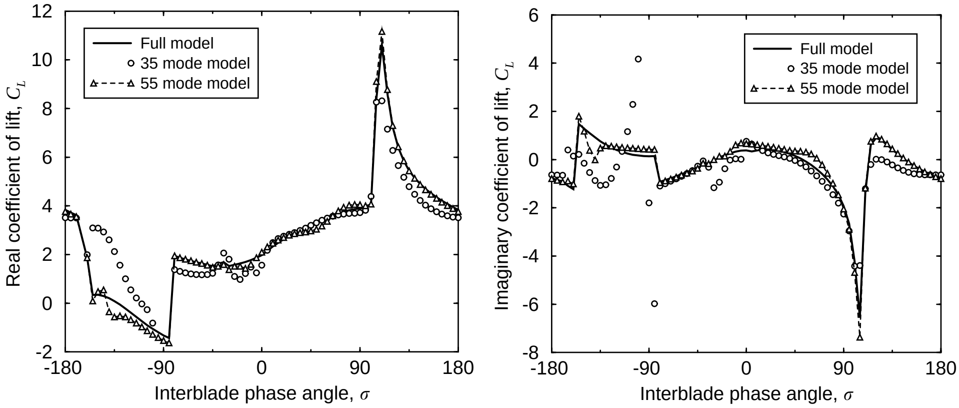

Fig. 10. Unsteady C_L obtained using the full model and a ROM at a Mach number of 0.9 and a reduced frequency k of 1.

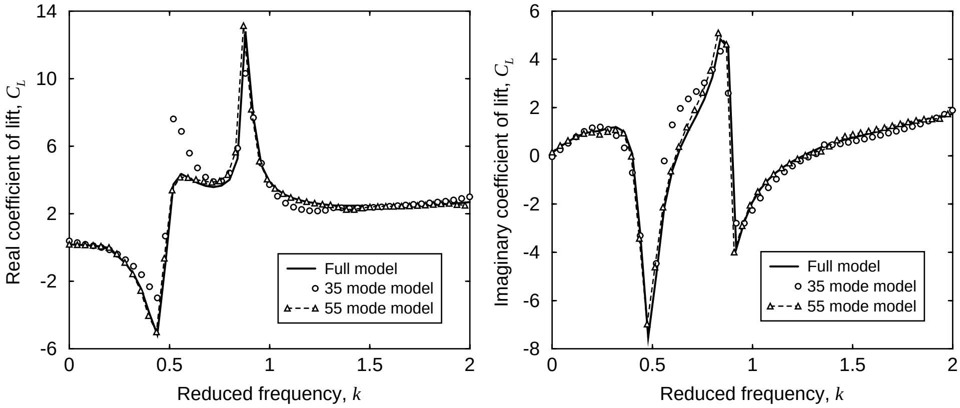

Fig. 11. Unsteady C_L obtained using the full model and a ROM at a Mach number of 0.9 and an interblade phase angle σ of 90°.

An interesting question posed when using reduced order modelling concerns the applicability of the same model reduction to different background flows. For inviscid flows, it has been recently shown (Epureanu et al., 2001a) that snapshots obtained at a given Mach numbers may be used with reasonable success at other Mach numbers. However, the effect of the viscosity (and especially the flow separation) leads to rapid and strong changes in the basis functions obtained for model reduction at distinct upstream far-field Mach numbers. Distinct from inviscid flows, Fig. 12 shows that the accuracy of a reduced order model constructed for a flow with an upstream far-field Mach number of 0.45 is not accurate when the snapshots used are computed for a flow with an upstream far-field Mach number of 0.5. Similar behavior is observed for various reduced frequencies as well as interblade phase angles, as shown in Fig. 13.

The pervasive existence of dominant modes the flow dynamics and their number may be observed by investigating the magnitudes of the eigenvalues of the correlation matrix obtained for various background flows. The eigenvalues shown in Fig. 14 have been scaled such that the largest eigenvalue has a magnitude of 1 for each upstream far-field Mach number. Fig. 14 shows that the dominant eigenvalues are the largest 35 eigenvalues in the case of subsonic flows and the largest 55 eigenvalues in the case of transonic flows.

An important factor in determining the number of modes in a reduced order model is the desired output of the model. Fig. 14 shows the magnitude of the eigenvalues of the correlation matrix for two separate types of correlation matrices: one is based on the potential and boundary layer variables, while the other is based on the pressure distribution on the airfoil surface. The criterion used for evaluating accuracy is most often the precision of the unsteady pressure predicted. In such cases, a correlation matrix based on the unsteady pressure is a more direct and precise indicator of the number of modes required. Nevertheless, the modes are computed using the POD method applied to the flow and boundary layer variables.

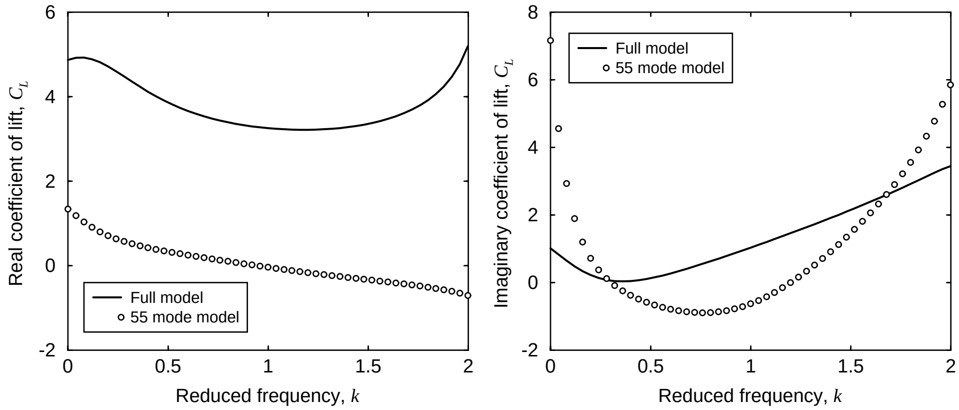

Fig. 12. Unsteady C_L obtained using the full model and a ROM at a Mach number of 0.9 and an interblade phase angle σ of 90° (snapshots at Mach 0.5; ROM results scaled down by a factor of 10).

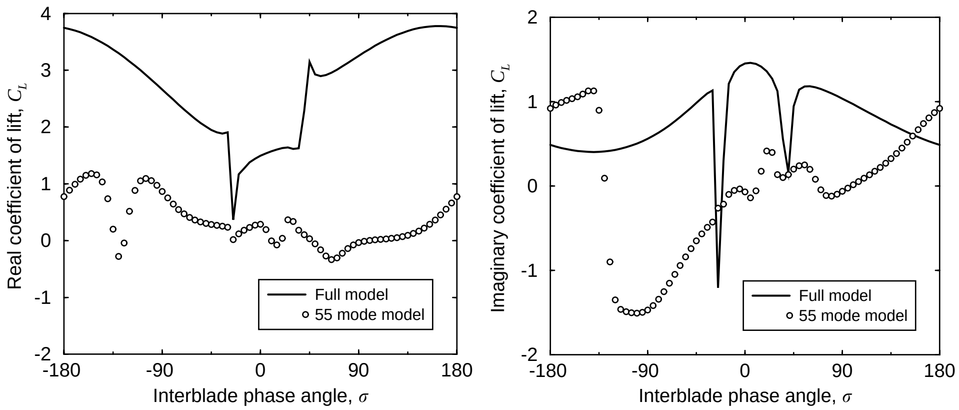

Fig. 13. Unsteady C_L obtained using the full model and a ROM at a Mach number of 0.9 and a reduced frequency k of 1 (snapshots at Mach 0.5; ROM results scaled down by a factor of 10).

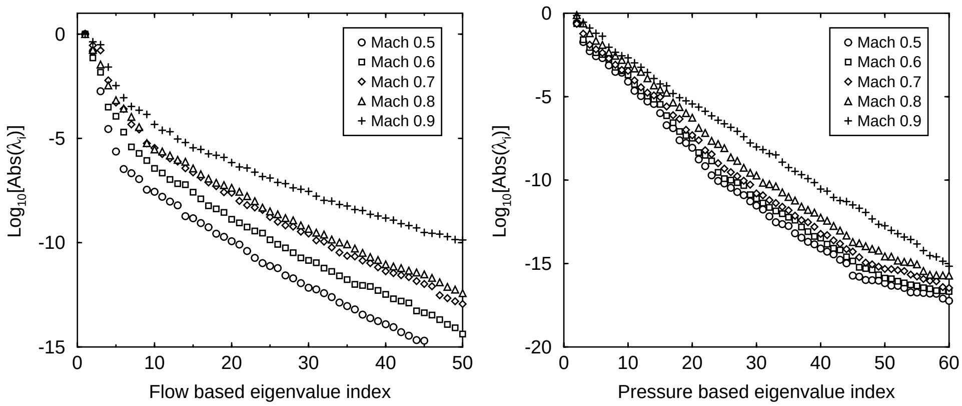

Fig. 14. Magnitudes of eigenvalues of the correlation matrices for various background flows. Eigenvalues are scaled such that the largest eigenvalue has a magnitude of 1 for each upstream far-field Mach number.

4. Discussion and conclusions

The unsteady flow created by a cascade of oscillating airfoils was calculated. The unsteadiness in the flow was assumed to be much smaller than the steady (nonlinear) flow. The dynamic perturbation about the nonlinear steady flow was linearized. An inviscid-viscous flow model was used. The inviscid flow was described by the full potential equation with appropriate upwinding in the transonic region. The viscous flow near the airfoils and in the wakes was described by an integral boundary layer model.

A frequency-domain model was constructed for the unsteady perturbation of the flow field. The proper orthogonal decomposition technique was then applied to this model to construct several reduced order models of subsonic and transonic flows. An intrinsic feature of the POD method is the need for computing snapshots. When the aerodynamic information per se is needed, then constructing the POD model takes about as much time as the original method. When the aerodynamic model is combined with a structural model, the time saved is orders of magnitude because one is using a POD model with less than 60 degrees of freedom instead of the original CFD model with approximately 6000 degrees of freedom. The purpose of the present analysis is to estimate the number of degrees of freedom required by accurate reduced order models by estimating the dimension of the inertial manifold of the flow dynamics. This estimate provide insightful physical understanding of the complex flow dynamics and provides useful information which may be used in a broad range of model reduction techniques.

A cascade of NACA-5506 airfoils has been investigated to show that reduced order models with only 35 degrees of freedom accurately predict the unsteady response of the full model with approximately 6000 degrees of freedom in the subsonic regime. Similarly, reduced order models with 55 degrees of freedom were shown to accurately predict the response of the full model in the transonic regime. Recently, it has been shown (Epureanu et al., 2001a) that the inviscid flow is characterized by a dimension of the inertial manifold of about 25 for *both* subsonic and transonic cases. Distinct from inviscid flows, the viscous flows show a different characteristic. The increase in the required number of degrees of freedom observed in the transonic viscous flows suggests that the dimension of the inertial manifold increases from approximately 35 to approximately 55. This increase is due to the presence of the shock and not the increase in the Mach number per se. The inertial manifold has a dimension of approximately 35 for a broad range of Mach numbers, from 0.5 to 0.8.

Previous studies have shown that 75 modes are required to accurately model transonic viscous flows (Epureanu et al., 2000) when approximately 100 snapshots are used for model reduction. In the present analysis, 700 snapshots were used and reduced order models of similar accuracy were constructed with only 55 modes. The difference in the number of modes indicates that the accuracy of the POD modes use for reduction is a critical factor in constructing reduced order models. The dimension of the manifold is about 55 in previous studies as well as in the present study. However, more modes are necessary—for the same accuracy—when inaccurate POD modes are used. Thus, when a smaller number of snapshots is used, it seems that the inertial manifold is larger when in fact it is not.

In certain applications, the proposed approach has several limitations. For example, modelling flows over a range of geometries and Mach numbers may not be accomplished. Also, the present analysis is only valid for small amplitude oscillations about a large magnitude steady state, and for flows with mild shocks. Nevertheless, the present analysis clarifies another important point, i.e., the dimension of the inertial manifold is about 35 for all subsonic flows of interest. However, *the shape of the dominant modes* is not the same. This fact is demonstrated by the results showing that significantly lower accuracy is obtained when modes obtained at a Mach number of 0.5 were used at a Mach number of 0.45. The change in the shape of the dominant modes seems to be rapid and substantial for the subsonic case discussed. Thus, the POD snapshots at a Mach number of 0.5 cannot be used at a Mach number of 0.45. Nevertheless, as the Mach number changes, the POD modes remain the same in number (for subsonic flows) although they change in shape.

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